

1 多倍長実数計算の実装の試み sin, cos, tan

$\sin(5^\circ) = 0.087155740$	
$\sin(5^\circ) = 0.087155742$	Windows 電卓
$\cos(5^\circ) = 0.996194699$	
$\cos(5^\circ) = 0.996194698$	Windows 電卓
$\tan(5^\circ) = 0.087488660$	
$\tan(5^\circ) = 0.087488663$	Windows 電卓
$\sin(8^\circ) = 0.139173097$	
$\sin(8^\circ) = 0.139173100$	Windows 電卓
$\cos(8^\circ) = 0.990268070$	
$\cos(8^\circ) = 0.990268068$	Windows 電卓
$\tan(8^\circ) = 0.140540830$	
$\tan(8^\circ) = 0.140540834$	Windows 電卓
$\sin(10^\circ) = 0.173648173$	
$\sin(10^\circ) = 0.173648177$	Windows 電卓
$\cos(10^\circ) = 0.984807754$	
$\cos(10^\circ) = 0.984807753$	Windows 電卓
$\tan(10^\circ) = 0.176326975$	
$\tan(10^\circ) = 0.176326980$	Windows 電卓
$\sin(30^\circ) = 0.499999980$	
$\sin(30^\circ) = 0.5$	Windows 電卓
$\cos(30^\circ) = 0.866025272$	
$\cos(30^\circ) = 0.866025403$	Windows 電卓
$\tan(30^\circ) = 0.577350333$	
$\tan(30^\circ) = 0.577350269$	Windows 電卓
$\sin(45^\circ) = 0.707106454$	
$\cos(45^\circ) = 0.707103232$	
$\cos(45^\circ) = 0.707106781$	Windows 電卓
$\tan(45^\circ) = 1.000004556$	
$\tan(45^\circ) = 1.000000000$	Windows 電卓

10° では 8 桁迄は正しく計算されるがこれを超えると精度は落ちてくる。30° では 6 桁迄、45° では 5 桁迄しか正しくない。

大きな角度に対しては例えば倍角公式を用いたり

$$\sin(2x + d) = 2 \sin(x) \cos(x) \cos(d) + \sin(d)(1 - 2 \sin^2(x)) \quad d = 0, 1$$

$$\sin(3x) = 3 \sin(x) - 4 \sin^3(x)$$

$$\sin(3x + d) = \sin(3x) \cos(d) + \sin(d) \cos(3x) \quad d = \pm 1$$

和の公式を利用するなり

$$\sin(30 + x) = 0.5 \cos(x) + (\sqrt{3}/2) \sin(x) \quad (0 \leq x < 15)$$

$$\cos(30 + x) = (\sqrt{3}/2) \cos(x) - 0.5 \sin(x) \quad (0 \leq x < 15)$$

$$\sin(45 + x) = 1/\sqrt{2}(\sin(x) + \cos(x))$$

$$\cos(45 + x) = 1/\sqrt{2}(\cos(x) - \sin(x))$$

しないとならないだろうがそのためには多倍長の実数の四則演算を実装することになる。

2 source

```
\makeatletter
% ----- \@@fdiv -----
\def\@@fdiv[#1]#2#3#4#5{% #1:計算桁数 #2/#3 = 整数部 #4 小数部 #5
  \edef\@@@nprec{#1}
  \edef\@@@x{#2}
  \edef\@@@y{#3}
  \@@@cnt=0%
  \edef\@@@ans{\empty}
%
% \@LDIV\@@@x\@@@y\@@@aaaa\@@@bbbb
\edef#4{\@@@aaaa}
%
% \@whilenum\@@@cnt<\@@@nprec\do{%
  \xdef\@@@x{\@@@bbbb0}\relax%
  \@LDIV\@@@x\@@@y\@@@aaaa\@@@bbbb%
  \xdef\@@@ans{\@@@ans\@@@aaaa}
  \@SUPZERO\@@@bbbb\@@@0\relax%
  \ifx\@@@0\NULL\edef\@@@bbbb{0}\fi%
  \advance\@@@cnt by 1%
}%
\edef#5{\@@@ans}
}%
\def\@fprecision{9}
\def\@fdiv{\@ifnextchar [{\@@fdiv}{\@@fdiv[\@fprecision]}}% default 9桁
%
% ----- \SinCos -----
\edef \DtoR{17453292}% 51994329576 /180*\BASE 度 ラジアン
% sin 係数
\edef \SinA{1000000000}% 0000000000 \BASE / 1! x
\edef \SinB{1666666666}% 6666666666 \BASE / 3! x^3
\edef \SinC{833333333}% 3333333333 \BASE / 5! x^5
\edef \SinD{198412}% 69841269841 \BASE / 7! x^7
\edef \SinE{2755}% 73192239858 \BASE / 9! x^9
\edef \SinF{25}% 05210838544 \BASE / 11! x^11
% cos 係数
\edef \CosA{5000000000}% 0000000000 \BASE / 2! x^2
\edef \CosB{416666666}% 6666666666 \BASE / 4! x^4
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\edef \CosC{1388888}% 8888888888 \BASE / 6! x^6
\edef \CosD{24801}% 58730158730 \BASE / 8! x^8
\edef \CosE{275}% 57319223985 \BASE / 10! x^10
\edef \CosF{2}% 08767569878 \BASE / 12! x^12
%-----
\newcount\ccnt
\edef\precision{9}% 小数点以下の桁数を9桁に設定
% ----- \SinCos -----
% sin x = x-x^3/3!+x^5/5!-[x^7/7!]
% cos x = 1-x^2/2!+x^4/4!-x^6/6!+[x^8/8!]
%
\def\SinCosTan#1#2#3#4{% #1= ,#2=sin ,#3=cos ,#4=tan
\edef\sita{#1}%
%
\ccnt=0
\edef\BASE{1}
\loop\ifnum\ccnt<9% 掛けた \BASE(9桁) を元に戻すため
\edef\BASE{\BASE0}
\advance\ccnt by 1
\repeat
\edef\BASE@\BASE{%
\edef\BASE@CosA{\BASE}% 1=x^0
%
\loop\ifnum\ccnt<27% 2*9+9
\edef\BASE{\BASE0}
\advance\ccnt by 1
\repeat
\edef\BASE@CosB{\BASE}%
%
\loop\ifnum\ccnt<36% 3*9+9
\edef\BASE{\BASE0}
\advance\ccnt by 1
\repeat
\edef\BASE@SinB{\BASE}%
%
\loop\ifnum\ccnt<45% 4*9+9
\edef\BASE{\BASE0}
\advance\ccnt by 1
\repeat
\edef\BASE@CosC{\BASE}%
%
\loop\ifnum\ccnt<54% 5*9+9
\edef\BASE{\BASE0}
\advance\ccnt by 1
\repeat
\edef\BASE@SinC{\BASE}%
%
\loop\ifnum\ccnt<63% 6*9+9
\edef\BASE{\BASE0}

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        \advance\ccnt by 1
\repeat
    \edef\BASE@CosD{\BASE}%
%
\loop\ifnum\ccnt<72%          7*9+9
    \edef\BASE{\BASE0}
    \advance\ccnt by 1
\repeat
    \edef\BASE@SinD{\BASE}%
%
\MUL\sitadtoR\rad%          x
\MUL\rad\rad\rrad%
    \MUL\rrad\CosA\cosxx%   x^2/2!
    \fdiv[\precision]\cosxx\BASE@CosB\ab
    \edef\cosxx{\a\b}
\MUL\rrad\rrad\xxxxrad%
    \MUL\xxxxrad\CosB\cosxxxx%   x^4/4!
    \fdiv[\precision]\cosxxxx\BASE@CosC\ab
    \edef\cosxxxx{\a\b}
\MUL\xxxxrad\rrad\xxxxxxrad%
    \MUL\xxxxxxrad\CosC\cosxxxxxx%   x^6/6!
    \fdiv[\precision]\cosxxxxxx\BASE@CosD\ab
    \edef\cosxxxxxx{\a\b}
%
\fdiv[\precision]\rad\BASE@\ab
\edef\sinx{\a\b}
%
\MUL\rrad\rad\rrad%
    \MUL\rrad\SinB\sinxxx%       x^3/3!
    \fdiv[\precision]\sinxxx\BASE@SinB\ab
    \edef\sinxxx{\a\b}
%
\MUL\xxxxrad\rad\xxxxxxrad%
    \MUL\xxxxxxrad\SinC\sinxxxxx%   x^5/5!
    \fdiv[\precision]\sinxxxxx\BASE@SinC\ab
    \edef\sinxxxxx{\a\b}
%
\MUL\xxxxxxrad\rad\xxxxxxxxrad%
    \MUL\xxxxxxxxrad\SinD\sinxxxxxx%   x^7/7!
    \fdiv[\precision]\sinxxxxxx\BASE@SinD\ab
    \edef\sinxxxxxx{\a\b}
%
\SUB\sinx\sinxxx\sin@val% x-x^3/3!
\ADD\sin@val\sinxxxx\sin@val% x-x^3/3!+x^5/5!
\SUB\sin@val\sinxxxxxx\sin@val% x-x^3/3!+x^5/5!-x^7/7!
\fdiv[9]\sin@val\BASE@\ab%
\edef#2{\a.\b}%
%
\SUB\BASE@CosA\cosxx\cos@val% 1-x^2/2!

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\ADD\cos@val\cosxxxx\cos@val% 1-x^2/2!+x^4/4!
\SUB\cos@val\cosxxxxx\cos@val% 1-x^2/2!+x^4/4!-x^6/6!
\fdiv[9]\cos@val\BASE@{a}\b
\edef#3{\a.\b}%
\fdiv[9]\sin@val\cos@val\tan@int\tan@dec% tan = sin/cos
\edef#4{\tan@int.\tan@dec}%
}%
\makeatother

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